



# Heat capacity of $\text{Ce}_{1-x}\text{La}_x\text{Cu}_4\text{Al}$ Kondo alloys

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## ABSTRACT

The temperature dependence of the specific heat  $C_p(T)$ , for  $\text{Ce}_{1-x}\text{La}_x\text{Cu}_4\text{Al}$  alloys has been studied. The specific heat has been analyzed considering the electronic, phonon and Schottky contributions. In comparison to  $\text{CeCu}_4\text{Al}$ , the substitution of Ce by La reduces the electronic specific heat coefficient  $\gamma$  values. At low temperatures  $\gamma$  value depends strongly on the temperature range used for the extrapolation and on the magnetic field. The scheme of the energy levels created by the crystal electric field (CEF) splitting has been determined from the Schottky anomaly. The values obtained for the energy of the levels are similar for all the compositions.

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## 1. Introduction

The intermediate valence, Kondo behaviour, spin fluctuations, heavy fermions and quantum critical point are most often observed in the Ce-based compounds. It is a direct consequence of the tendency of the Ce 4f states to hybridize strongly with the conduction electrons.

The  $\text{CeCu}_5$  compound was identified as a Kondo lattice compound exhibiting antiferromagnetism below 4 K and a high  $\gamma$  value ( $100 \text{ mJ mol}^{-1} \text{ K}^{-2}$ ) of the specific heat [1]. The substitution of Cu by Al and Ga increases the average electron density per atom and therefore, a significant change of the Kondo lattice behaviour is expected [2–7].  $\text{CeCu}_4\text{Al}$  is a heavy fermion compound and it is the derivate of  $\text{CeCu}_5$ . It is paramagnetic and follows the Curie–Weiss law with the effective magnetic moment  $\mu_{\text{eff}} = 2.53 \mu_B/\text{f.u.}$  and the paramagnetic Curie temperature  $\theta_p = -10 \text{ K}$  [2]. The estimated value of the electronic coefficient  $\gamma$  is about  $2.2 \text{ J mol}^{-1} \text{ K}^{-2}$ .

Recently the effect of La dilution in  $\text{CeCu}_4\text{Al}$  was investigated, mainly in relation to the electrical resistivity and magnetic susceptibility properties [8,9]. In all of the alloys studied, no evidence of a phase transition was observed down to the lowest measuring temperature of 2 K [8]. Susceptibility measurements give effective magnetic moments close to the  $\text{Ce}^{3+}$  ion value. The analysis of the magnetic resistivity at low temperatures revealed that only  $\text{CeCu}_4\text{Al}$  and  $\text{Ce}_{0.95}\text{La}_{0.05}\text{Cu}_4\text{Al}$  show a typical of the Kondo lattice maximum in  $\rho(T)$  [8]. This behaviour is associated with the interplay of the dilution effects and the volume effects upon

alloying. For all the Ce-containing alloys in this series  $\rho_m(T) \sim -\ln T$  for temperature increase above 2 K, as is expected in the case of the incoherent Kondo scattering. In the low temperature region, magnetoresistivity exhibits a negative magnetic field dependence due to the Kondo effect. Magnetoresistivity measurements on the  $\text{Ce}_{1-x}\text{La}_x\text{Cu}_4\text{Al}$  alloys has been analyzed basing on the calculations by Schlottmann for the Bethe-ansatz in the frames of the Coqblin–Schrieffer model [9]. Kondo temperature  $T_K$  is decreasing with the content of La, which is probably due to the unit cell volume increase.

In this paper we present the specific heat results for the  $\text{Ce}_{1-x}\text{La}_x\text{Cu}_4\text{Al}$  compounds that are analyzed in terms of the different contributions.

## 2. Experimental details

The preparation of the polycrystalline samples of the solid solutions  $\text{Ce}_{1-x}\text{La}_x\text{Cu}_4\text{Al}$  ( $0.0 \leq x \leq 1$ ) was described in the previous paper [8]. The hexagonal  $\text{CeCu}_5$ -type structure was confirmed by the powder x-ray diffraction technique. The unit cell volume  $V$  of  $\text{Ce}_{1-x}\text{La}_x\text{Cu}_4\text{Al}$  increases with the increase of the La concentration [8].

Heat capacity measurements were carried out by the PPMS commercial device (Quantum Design) in the temperature range 1.9–300 K by the relaxation method using the two- $\tau$  model. The thermometers have been calibrated in various magnetic fields, including the fields used in the present studies.

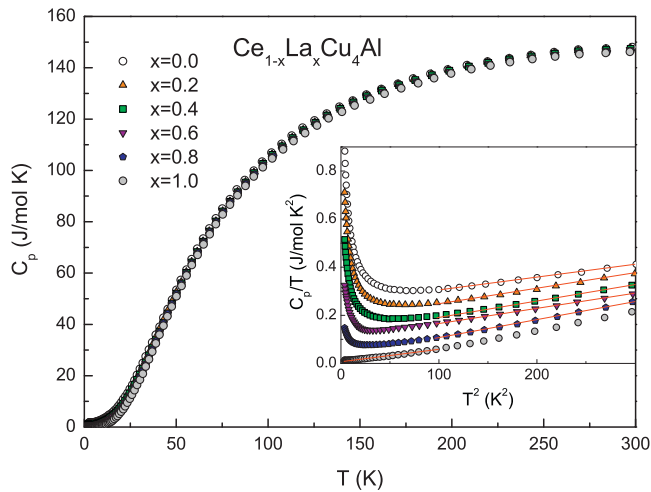
## 3. Results and discussion

Heat capacity of the studied compounds can be defined in a general way as a sum of the respective contributions:

$$C_p = C_{el} + C_{ph} + C_{mag} \quad (1)$$

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**Fig. 1.** Temperature dependence of the heat capacity of the studied compounds up to 300 K in zero magnetic field. The inset displays the heat capacity data at the low temperature of the  $C_p/T$  vs.  $T^2$  dependence for  $Ce_{1-x}La_xCu_4Al$  as a function of  $x$  used to determine the electronic specific heat coefficient  $\gamma$ .

The first term represents the electronic contribution in the form  $C_{el} = \gamma T$ , where  $\gamma$  is the Sommerfeld coefficient that is proportional to the density of states at the Fermi level. Second term describes the lattice contribution due to the lattice vibrations and plays a significant role at higher temperatures. A good description of the phonon spectrum can be obtained in the frames of the Debye and Einstein models. The last term corresponds to the magnetic part of  $C_p(T)$  and is connected with the split of the ground state of the magnetic ions to the energy levels by the crystalline electric field (CEF). The thermal population of the energy levels leads to a maximum in the specific heat known as the Schottky anomaly.

Fig. 1 shows the temperature dependence of the heat capacity  $C_p(T)$  of  $Ce_{1-x}La_xCu_4Al$  in the temperature range 1.9–300 K in zero magnetic field. This measurement does not show any real sign of the magnetic order down to 1.9 K. The electronic part of the specific heat is expressed in a linear way as  $C_{el} = \gamma T$  and at adequately low temperatures ( $T \ll \Theta_D$ ), when the lattice contribution of optic branches is negligible the Debye model can be written in simplified form as  $C_p(T) = \gamma T + \beta T^3$ , where  $\beta$  is related to the Debye temperature by  $\Theta_D = (12\pi^4 R n / 5\beta)^{1/3}$ . The value of the electronic coefficient of the specific heat  $\gamma$  has been taken as the extrapolation of the linear part of the  $C_p/T$  vs.  $T^2$  curves at low temperatures (inset in Fig. 1). The upturn in  $C_p/T$  observed at low temperatures is characteristic of many heavy fermion and moderate heavy fermion compounds. The substitution of Ce by La reduces the electronic heat capacity coefficient  $\gamma$  values from 254 mJ/mol K<sup>2</sup> for  $CeCu_4Al$  to 9.2 mJ/mol K<sup>2</sup> for  $LaCu_4Al$  (Table 1). The value of  $\gamma$  for  $LaCu_4Al$  is similar to the value obtained by Bauer et al. [7] (11 mJ/mol K<sup>2</sup>) and Toliński et al. [3] for  $YCu_4Al$  (9.13 mJ/mol K<sup>2</sup>). Since  $\gamma$  is proportional to the density of states at the Fermi level, a reduction in the value of  $\gamma$  for  $Ce_{1-x}La_xCu_4Al$  indicates a decrease in the density of states. As the values of the atomic mass of La and Ce are nearly

the same, the Debye temperatures  $\Theta_D$  are expected to be similar. Therefore,  $LaCu_4Al$  is a good candidate for the non-magnetic analog of  $CeCu_4Al$ . The values of  $\Theta_D$  obtained from the simplified Debye model are approximating for all the investigated compounds (Table 1).

Heat capacity in the whole temperature range can be expressed by the standard Debye formula with the addition of the electronic part:

$$C_p(T) = \gamma T + 9NR \left( \frac{T}{\Theta_D} \right)^3 \int_0^{\Theta_D/T} \frac{x^4 e^x dx}{(e^x - 1)^2} \quad (2)$$

where  $N = 6$  is the number of the atoms in the formula unit,  $R$  is the gas constant and  $x = \hbar\omega/k_B T$ . However, this approach cannot fully describe the specific heat data of  $LaCu_4Al$  due to the lack of the optical modes which are significant in higher temperatures. The most realistic description of the phonon part of the specific heat, especially at low temperatures has been achieved by considering the splitting of the phonon spectrum into the acoustic and optical branches. The optical modes are given by the Einstein formula:

$$C_{Ei}(T) = R \sum_{i=1}^{15} \left( \frac{\Theta_{Ei}}{T} \right)^2 \frac{e^{\Theta_{Ei}/T}}{(e^{\Theta_{Ei}/T} - 1)^2} \quad (3)$$

where  $\Theta_{Ei}$  is the characteristic Einstein temperature for each optical branch.

For nonmagnetic  $LaCu_4Al$  the phonon spectrum consists of 3-acoustic branches which are described by the Debye model and 15-optical branches attributed to the Einstein model. The optical branches were divided to three groups with different degeneracy  $n_{Ei}$ : (5–9–1). The isobaric specific heat of  $LaCu_4Al$  was fitted using both the Debye and Einstein models together with their anharmonic correction coefficients  $\alpha_D$  and  $\alpha_{Ei}$ , which accounts for the discrepancy between the isobaric and the isochoric specific heat [10,11]:

$$C_p(T) = \gamma T + R \left\{ \frac{9}{1 - \alpha_D T} \left( \frac{T}{\Theta_D} \right)^3 \int_0^{\Theta_D/T} \frac{x^4 e^x dx}{(e^x - 1)^2} \right\} + R \left\{ \sum_{i=1}^{15} \frac{1}{1 - \alpha_{Ei} T} \left( \frac{\Theta_{Ei}}{T} \right)^2 \frac{e^{\Theta_{Ei}/T}}{(e^{\Theta_{Ei}/T} - 1)^2} \right\} \quad (4)$$

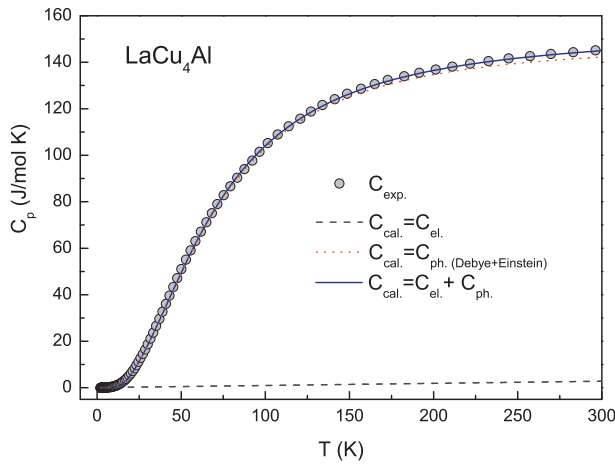
The best fit has been obtained with the following parameters:  $\Theta_D = 244$  K and  $\Theta_{Ei} = 116$  K, 253 K, 420 K, with their anharmonic coefficients:  $\alpha_D = 1.3 \times 10^{-4}$  and  $\alpha_{Ei} = 1.6 \times 10^{-5}$ ,  $1 \times 10^{-5}$ ,  $1.1 \times 10^{-5}$ , respectively. The experimental heat capacity data in the  $C_p(T)$  representation together with the calculated contributions  $C_e$  and  $C_{ph}$  are shown in Fig. 2 and the same data in  $C_p/T(T)$  representation together with the fitting parameters are presented in Fig. 3.

To analyze the magnetic part of the specific heat of  $Ce_{1-x}La_xCu_4Al$  we used the specific heat of the nonmagnetic  $LaCu_4Al$  analog to get information about the electronic and phonon contributions. The magnetic contribution to the specific heat of the  $CeCu_4Al$  sample is depicted in Fig. 4.  $C_{mag}/T$  consists of two main contributions. The first one is the hump at about 20 K, which is a consequence of the crystal electric field effects (CEF). The second one is the steep increase at the lowest temperatures, which is probably developed by the Kondo interactions. With the increase of the La concentration this specific heat anomalies remain roughly at the same position. The hexagonal symmetry splits the sixfold degenerate state of  $Ce^{3+}$  with  $J = 5/2$  into three doublets with the energy gaps  $\Delta_1$  and  $\Delta_2$  from the ground state. The contribution to the magnetic specific heat, connected with the CEF effects was fitted

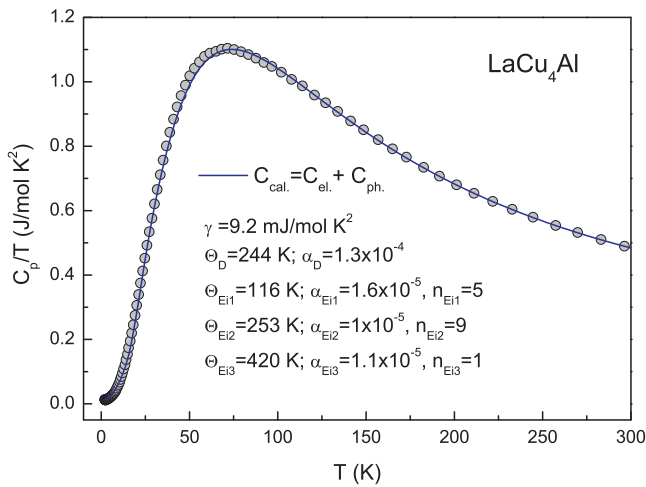
**Table 1**

Parameters obtained from specific heat analysis: electronic specific heat coefficients  $\gamma$ , Debye temperatures  $\Theta_D$  and the values of the parameters of the formula  $C_p/T = \gamma_0 \ln(T_0/T)$ .

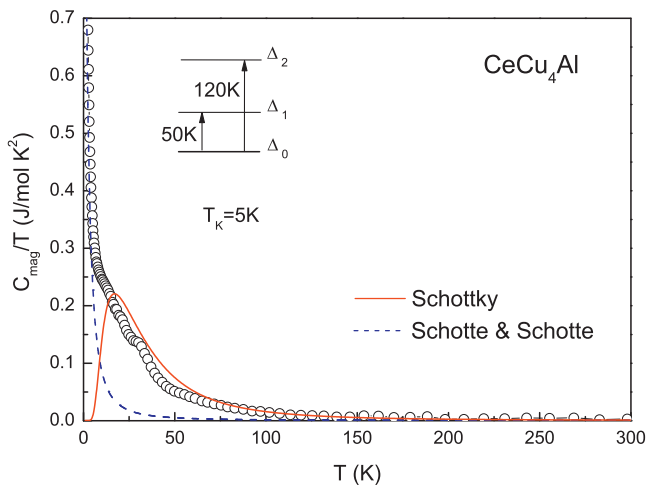
$x$	$\gamma$ (mJ/mol K <sup>2</sup> )	$\Theta_D$ (K)	$\gamma_0$ (J/mol K <sup>2</sup> )	$T_0$ (K)
0.0	254	285	0.777	6.10
0.2	190	270	0.579	6.35
0.4	135	267	0.431	6.57
0.6	106	262	0.260	6.69
0.8	36	257	0.111	7.12



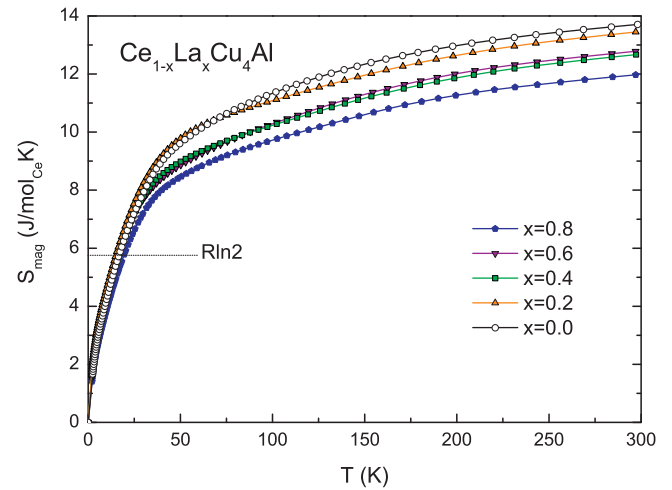
**Fig. 2.** Experimentally obtained heat capacity data of LaCu<sub>4</sub>Al together with the calculated values of the electronic  $C_e$  and the phonon part  $C_{ph}$ .



**Fig. 3.** Temperature dependence of specific heat displayed as  $C_p/T$  for LaCu<sub>4</sub>Al. Solid line represents fit to the formula (Eq. (4)) given in the text and together with all the fitting parameters.



**Fig. 4.** Example of the analysis of the magnetic specific heat with the Schottky (Eq. (5)) and the Schotte & Schotte (Eq. (6)) formulas.



**Fig. 5.** Temperature dependence of the magnetic entropy.

with the Schottky formula [12,13]:

$$C_{Sch}(T) = \frac{R}{T^2} \left[ \frac{\sum_{i=0}^{n-1} \Delta_i^2 e^{-\Delta_i/T}}{\sum_{i=0}^{n-1} e^{-\Delta_i/T}} - \left( \frac{\sum_{i=0}^{n-1} \Delta_i e^{-\Delta_i/T}}{\sum_{i=0}^{n-1} e^{-\Delta_i/T}} \right)^2 \right] \quad (5)$$

and the energy levels  $\Delta_i$  (assuming  $\Delta_0 = 0$  K) of the Ce<sup>3+</sup> ground state multiplet have been determined (Fig. 4). For CeCu<sub>4</sub>Al the first CF energy level lies about 50 K and second one about 120 K above the ground state doublet. Energy schemes in a similar values range were deduced from the previous specific heat measurements ( $\Delta_1 = 65$  K,  $\Delta_2 = 85$  K or  $\Delta_1 \approx \Delta_2 \approx 90$  K) [3,7]. We have performed a similar analysis for other samples. The magnetic part of specific heat analysis indicates that the crystal field parameters are almost independent of the La concentration.

The growth of  $C_{mag}/T$  at low temperatures has been estimated by the Schotte & Schotte formula [14]:

$$\frac{C_{Kondo}}{T} = \frac{k_B N_A T_K}{\pi T^2} \left( 1 - \frac{T_K}{2\pi T} \psi' \left( \frac{1}{2} + \frac{T_K}{2\pi T} \right) \right) \quad (6)$$

where  $\psi'$  is the derivative of the Digamma function and Kondo temperature  $T_K$  is the only fitting parameter. The received values of  $T_K$  for CeCu<sub>4</sub>Al is about 5 K and is in good agreement with values obtained from magnetoresistivity ( $T_K = 6$  K) [9] and also other measurements [2,8].

The analysis presented in Fig. 4 shows that the use of the La-based analog to get the magnetic part of the heat capacity provides slightly different set of parameters (energy levels, Kondo temperature) than in the case of the Y-based analog used in our previous studies [3]. The inelastic neutron scattering has only indicated that one of the excited levels should be around 90 K for CeCu<sub>4</sub>Al [3,7].

The integration of the  $C_{mag}/T$  vs.  $T$  dependence provides to the value of the magnetic entropy, which is related to the degrees of freedom by the expression  $S_{mag} = R \ln(2J+1)$ . Using the formula:

$$S_{mag} = \int_0^T \frac{C_{mag}(T)}{T} dt \quad (7)$$

the temperature dependence of the magnetic entropy was calculated (Fig. 5). For a doublet ground state  $R \ln 2 = 5.76$  J/mol K. This value of  $S_{mag}$  is reached in 100% at about 15 K for all the Ce<sub>1-x</sub>La<sub>x</sub>Cu<sub>4</sub>Al compositions. The reduction of the magnetic entropy at higher temperatures can be ascribed to the Kondo effect.

In Fig. 6 we present results of the specific heat of Ce<sub>1-x</sub>La<sub>x</sub>Cu<sub>4</sub>Al ( $x = 0.2, 0.4, 0.6, 0.8$ ) in zero magnetic field and  $H = 9$  T. The semi-log

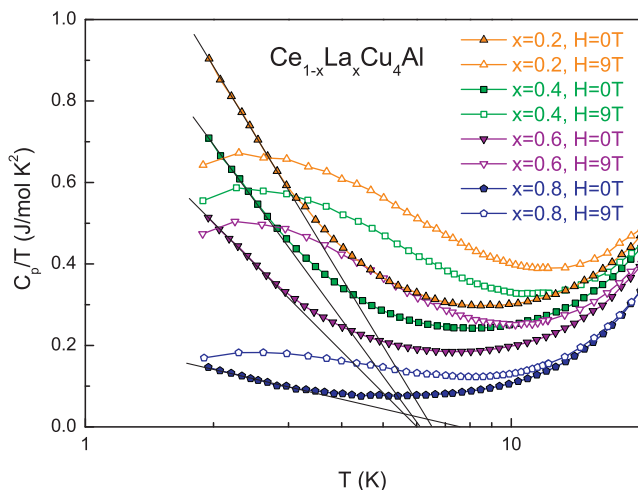


Fig. 6. The plot of  $C_p/T$  for  $\text{Ce}_{1-x}\text{La}_x\text{Cu}_4\text{Al}$  in zero magnetic field and  $H=9\text{ T}$ .

plot in Fig. 6 emphasizes the logarithmic increase of the electronic specific heat coefficient below 3 K in zero magnetic field. This type of dependence in low-temperature part of  $C_p(T)/T$  is similar to the non-Fermi-liquid (NFL) behaviour [15,16], which may be attributed to the proximity of the magnetic phase transition ( $T_N \approx 0.5\text{ K}$  for  $x=0.0$ ) [17]. Nevertheless, the measurements down to mK range are needed to confirm this presumptions. The zero magnetic field experimental data was fitted a straight line corresponding to the  $C_p/T = \gamma_0 \ln(T_0/T)$  and the obtained parameters  $\gamma_0$ ,  $T_0$  are presented in Table 1. The applied field of  $H=9\text{ T}$  suppresses the rapid rise at low temperatures. Generally, the specific heat of  $\text{Ce}_{1-x}\text{La}_x\text{Cu}_4\text{Al}$  behaves in high magnetic fields in a manner typical of Fermi liquid.

#### 4. Conclusions

The linear specific heat coefficient decreases with the increasing La concentration indicating decrease in the density of states at the Fermi level. At low temperatures  $\gamma$  value depends strongly on the temperature range used for the extrapolation and on the magnetic field. The anomaly in  $C_{mag}/T$  near 20 K is described reasonably well by the Schottky contribution and the Schotte & Schotte formula. The

specific heat analysis indicates that the crystal field parameters are practically independent of the La content. The Kondo temperature of  $\text{CeCu}_4\text{Al}$  estimated from the magnetic part of specific heat by using Schotte & Schotte formula takes a value of about 5 K, which is in good agreement with values obtained from other measurements. The magnetic entropy, calculated from the excess specific heat due to the cerium 4f electrons reaches a value of  $R \ln 2$  per Ce atom (expected for a doubly degenerated ground state) around 15 K. The reduction and not large differences of the magnetic entropy at higher temperatures can be caused by the Kondo effect. At low temperatures ( $T \leq 3\text{ K}$ ), the ratio  $C_p/T$  for  $\text{Ce}_{1-x}\text{La}_x\text{Cu}_4\text{Al}$  shows a logarithmic dependence for  $H=0\text{ T}$ .

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#### References

- [1] E. Bauer, D. Gignoux, D. Schmitt, K. Winzer, J. Magn. Magn. Mater. 69 (1987) 158.
- [2] A. Kowalczyk, T. Toliński, M. Reiffers, M. Pugaczowa-Michalska, G. Chełkowska, J. Phys.: Condens. Matter 20 (2008) 255252.
- [3] T. Toliński, A. Hoser, S. Rols, A. Kowalczyk, A. Szalaferek, Solid State Commun. 149 (2009) 2240.
- [4] A. Kowalczyk, M. Pugaczowa-Michalska, T. Toliński, Phys. Stat. Sol. (b) 242 (2005) 433.
- [5] E. Bauer, E. Gratz, N. Pillmayr, Solid State Commun. 62 (1987) 271.
- [6] E. Bauer, N. Pillmayr, E. Gratz, D. Gignoux, D. Schmitt, K. Winzer, J. Kohlmann, J. Magn. Magn. Mater. 71 (1988) 311.
- [7] E. Bauer, N. Pillmayr, H. Muller, J. Kohlmann, K. Winzer, J. Magn. Magn. Mater. 90–91 (1990) 411.
- [8] M. Falkowski, A. Kowalczyk, T. Toliński, Solid State Commun. 150 (2010) 1548.
- [9] M. Falkowski, A. Kowalczyk, T. Toliński, Intermetallics 19 (2011) 433.
- [10] C.A. Martin, J. Phys.: Condens. Matter 3 (1991) 5967.
- [11] P. Svoboda, P. Javorský, M. Diviš, V. Sechovský, F. Honda, G. Oomi, A.A. Menovsky, Phys. Rev. B 63 (2001) 212408.
- [12] P. Svoboda, J. Vejpravová, N.-T.H. Kim-Ngan, F. Kaysel, J. Magn. Magn. Mater. 272 (2004) 595.
- [13] T. Toliński, A. Kowalczyk, A. Szewczyk, M. Gutowska, J. Phys.: Condens. Matter 18 (2006) 3435.
- [14] K.D. Schotte, U. Schotte, Phys. Lett. A 55 (1975) 38.
- [15] H.v. Löhneysen, J. Phys.: Condens. Matter 8 (1996) 9689.
- [16] F.C. Ragel, P. de, V. du Plessis, A.M. Strydom, J. Phys.: Condens. Matter 21 (2009) 046008.
- [17] E. Bauer, E. Gratz, J. Kohlmann, K. Winzer, D. Gignoux, D. Schmitt, Z. Phys.: B Condens. Matter. 80 (1990) 263.